1. *The sum of the reciprocals of a set of n different positive integers is equal to one. If $n = 3$ show that there is only one such set and find it. Find such a set for any value of $n > 3$.*

2. *Given any real number $a \neq -1$, the sequence $\{x_i\}_{i=1}^{\infty}$ is defined by $x_1 = a$, and $x_{n+1} = x_n^2 + x_n$ for all $n \geq 1$. Let $y_n = 1/(1 + x_n)$. If S_n is the sum, and P_n is the product of the first n terms of the sequence $\{y_i\}_{i=1}^{\infty}$, prove that $aS_n + P_n = 1$ for all n.*

3. *Inside a cube of side 15 units there are 11000 given points. Prove that there exists a sphere of unit radius containing at least six of the given points.*

4. *Find all functions $f : \mathbb{R} \mapsto [1, \infty)$ which are differentiable on $\mathbb{R}$ and satisfy*

$$\int_1^{f(x)} e^{u^2}\, du = \int_0^x \frac{u\, du}{f(u)}, \quad x \in \mathbb{R}.$$

5. *Determine all functions $f \in \mathbb{C}^2[0, 1]$ such that*

(i) $f(0) = f'(0) = 1$;

(ii) $f''(x) \geq 0$, *for any* $x \in (0, 1)$;

(iii) $\int_0^1 f(x)\, dx = 3/2$.

6. *Find the largest real number K (independent of a, b, and c) such that the inequality*

$$a^2 + b^2 + c^2 > K(a + b + c)^2$$

holds for the lengths a, b, and c of the sides of any obtuse-angled *triangle.*

7. *Find all functions $f \in \mathbb{C}^4(\mathbb{R})$ such that, for all $x \in \mathbb{R}$, $f(x) \leq 0$ and $f^{(4)}(x) \geq 0$.*